## Launch of the ACE satellite

Canadä'

Intro. to Retrievals

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## Topics for Today

- Cross-section spectra
- Example spectral detector
- A simple spectrometer
- Instrument characterization
- Spectral fitting
- Sample data
- More advanced techniques
- Conclusions



# "Two-component Instrument" <br> 1. EG\&G Reticon Detector 

Detector: 1 inch long 1000 elements 0.001 inch each
$\qquad$ 0.1 inch

1 inch
Quantum efficiency peak 0.8 at $\sim 750 \mathrm{~nm}$
still useful $\sim 0.3$ down to 285 nm
Randomly Addressable
operate at short integration times integration time to fit light intensity
2. Concave Holographic Grating

American Holographic
Grating First-Order Efficiency ~0.5

## Diode-array Spectrometer



$$
\begin{aligned}
& \mathrm{dA}=1 \cdot \mathrm{w} \\
& \text { where } 1=\text { slit length } \\
& \text { and } \mathrm{w}=\text { slit width }
\end{aligned}
$$



## RETICON Detector

Pixels are charged to a reference level ${ }_{\circ}^{\circ}$ \& isolated by transmission gates Light causes the capacitance of the photodiodes to discharge
The number of electrons to recharge the pixel is measured
The result digitized and stored by computer
Charge is shared between pixels


Laserscans Single-Brewer stray light rejection is $10^{-5}-10^{-4}$ as measured by scanning 325 nm HeCd laserline



## Beer's Law

## Beer's Law

[note the changes from the handout]
(e.g.:

Photons / s / m ${ }^{2}$ )
$d \mathrm{l}(\lambda)=\mathrm{I}(\lambda) \sigma(\lambda) \rho \mathrm{ds}==\mathrm{I}(\lambda) \mathrm{d} \mathrm{T}$
Cross-section [m²]
$\sigma$ - cross-section [ $\mathrm{m}^{2}$ ]
$\mathrm{dI} / \mathrm{I}=-\mathrm{dT}$ Integrated: $\mathrm{I}=\mathrm{I}_{0} \mathrm{e}^{-\mathrm{T}} \rho$ - number density $\left[\mathrm{m}^{-3}\right]$
ds - differential length [m]
Or $T=\log \left[I_{0}(\lambda)\right]-\log [I(\lambda)] \quad T$ - optical depth
Where $\mathrm{I}_{0}$ is incident intensity

## Notation

$\mathrm{j}<\mathrm{N}_{\mathrm{c}}$
$\sum x_{j} y_{j} \quad$ Sum of $x_{0} y_{0}+x_{1} y_{1}+\ldots+x_{N-1} y_{N-1}$ Also $x_{j} y_{j}$ with repeated subscript convention
$j=0$ (sometimes called the Einstein convention)

Covariance

$$
\sigma_{\mathrm{jk}}=\sum\left(\mathrm{x}_{\mathrm{ij}}-\overline{\mathrm{x}}_{\mathrm{j}}\right)\left(\mathrm{x}_{\mathrm{ik}}-\bar{x}_{\mathrm{k}}\right)
$$

With mean:

$$
\bar{x}_{j}=\frac{1.0}{N_{i}} \sum_{i=0}^{i<N_{i}} x_{i j}
$$

But frequently $\mathrm{x}_{\mathrm{i}}$ and $\mathrm{x}_{\mathrm{j}}$ are considered uncorrelated and $\sigma$ diagonal

## Spectral Fitting

$$
I\left(\lambda_{i}\right)=\int I(\lambda) S\left(\lambda-\lambda_{i}\right) d \lambda \quad I(\lambda)-\text { light intensity }
$$

$\lambda_{i}$ effective wavelength of pixel $i$

$$
\begin{array}{ll}
C\left(\lambda_{i}\right)=R_{i} I\left(\lambda_{i}\right) & \begin{array}{l}
S(\lambda)-\text { slit function } \\
\\
\\
R_{i}-\text { response of pixel } i
\end{array} \\
I(\lambda)=I_{0}(\lambda) \exp [-\tau(\lambda)] & \begin{array}{l}
I_{0}(\lambda)-\text { source intensity }
\end{array} \\
\tau(\lambda)=\tau_{c}(\lambda)+\tau_{R}(\lambda)+\tau_{a}(\lambda) & \begin{array}{l}
c-\text { constitul depth }
\end{array} \\
& \text { R-Rayleigh (air) } \\
\text { Clearly, since: } & \text { a- aerosol (particles) } \\
\exp [-T(\lambda)]=\exp \left[-\tau_{c}(\lambda)\right] \exp \left[-\tau_{R}(\lambda)\right] \exp \left[-\tau_{a}(\lambda)\right] \\
& =\exp \left\{-\left[\tau_{c}(\lambda)+\tau(\lambda)_{R}+\tau(\lambda)_{a}\right]\right\}
\end{array}
$$

## Absorbers

$\tau_{c}(\lambda)=\sum_{j=0}^{j<N_{c}} \sigma_{j}(\lambda) \eta_{j}$
$\eta_{j}=\int_{s} \rho_{j}(s) d s$
$I\left(\lambda_{i}\right)=\int I_{o}(\lambda) \exp \left[-\sigma_{j}(\lambda) \eta_{j}\right] S\left(\lambda-\lambda_{i}\right) d \lambda$ sum over $j$

One approach to fitting the spectrum is to minimize The log-difference between the model and the measurements (m):

$$
\underset{\operatorname{SE} E}{ }=\sum_{i=0}^{i<N_{\lambda}}\left\{\log \left[I m\left(\lambda_{i}\right)\right]-\log \left[I\left(\lambda_{i}\right)\right]\right\}^{2}
$$

## Solution

Let $L\left(\lambda_{i}\right)==\log \left[I\left(\lambda_{i}\right)\right]$ Then the square difference becomes: $\quad i<N_{\lambda}$

$$
S E=\sum_{i=0}^{n}\left[L^{m}\left(\lambda_{i}\right)-L\left(\lambda_{i}\right)\right]^{2}
$$

SE depends on the amount of the various absorbers.
An estimate of the amounts can be done by minimizing SE with respect to the different constituents.

$$
\frac{\partial S E}{\partial \eta_{c}}=\sum_{i=0}^{i<N_{\lambda}} 2 \frac{\partial L\left(\lambda_{i}\right)}{\partial \eta_{c}}\left[L^{m}\left(\lambda_{i}\right)-L\left(\lambda_{i}\right)\right]==0.0
$$

At some point there will be a set of $\eta_{c}$ producing a $\log$ spectrum $L\left(\lambda_{i}\right)$ which is not the best fit to $L^{m}\left(\lambda_{i}\right)$

## Equation

Assume $\Delta L\left(\lambda_{i}\right)=\left[L^{m}\left(\lambda_{i}\right)-L\left(\lambda_{i}\right)\right]$

$$
\frac{\partial S E}{\partial \eta_{c}}=\sum_{i=0}^{i<N_{\lambda}} \frac{\partial L\left(\lambda_{i}\right)}{\partial \eta_{c}} \Delta L\left(\lambda_{i}\right) \text { near } 0.0
$$

Make a small displacement in $\eta_{c}\left(\Delta \eta_{c}\right)$ to make the sum smaller:

$$
\begin{gathered}
\delta L\left(\lambda_{i}\right)=\frac{\partial L\left(\lambda_{j}\right)}{\partial \eta_{k}} \Delta n_{k} \\
\sum_{i=0}^{i<N_{\lambda}} \frac{\partial L\left(\lambda_{i}\right)}{\partial \eta_{c}}\left[\Delta L\left(\lambda_{i}\right)-\frac{\partial L\left(\lambda_{i}\right)}{\partial \eta_{k}} \Delta \eta_{k}\right]==0.0
\end{gathered}
$$

## Matrix

$\sum_{i=0}^{i<N_{\lambda}} \frac{\partial L\left(\lambda_{i}\right)}{\partial \eta_{c}} \frac{\partial L\left(\lambda_{i}\right)}{\partial \eta_{k}} \Delta \eta_{k}=\sum_{i=0}^{i<N_{\lambda}} \frac{\partial L\left(\lambda_{i}\right)}{\partial \eta_{c}} \Delta L\left(\lambda_{i}\right)$
Let $\quad K_{c k}=\sum_{i=0}^{i<N_{\lambda}} \frac{\partial L\left(\lambda_{i}\right)}{\partial \eta_{c}} \frac{\partial L\left(\lambda_{i}\right)}{\partial \eta_{k}}$
and $\quad Y_{C}=\sum_{i=0}^{i<N_{\lambda}} \frac{\partial L\left(\lambda_{i}\right)}{\partial n_{c}} \Delta L\left(\lambda_{i}\right)$

$$
\mathrm{K}_{\mathrm{ck}} \Delta n_{\mathrm{k}}=\mathrm{Y}_{\mathrm{c}} \quad \Delta \boldsymbol{n}=\mathbf{K}^{-1} \mathbf{Y}
$$

## Raw Data <br> Vis Spectrometer

The observing strategy attempts to keep the detector wells full...


## Real Data - Counts/s



After dark count and stray light correction, scale to counts per second...

Note: Axes are shifted to line up the brightest spectra in the UV and Visible. All others line up because of the high linearity of the systems.

## ETS \& Fractional Uncertainty



## Optical Depth



A mean reference spectrum from 15 high-sun spectra is used to estimate wavelength-dependent optical depth

These apparent optical depth spectra are fitted to determine constituent surface densities (slant columns)

## $\mathrm{n}^{\text {th }}$ Iteration, Analytic Derivative

$$
\eta_{k}{ }^{n+1}=\eta_{k}{ }^{n}+\Delta \eta_{k}
$$

Remember?
(This is where the magic happens)

$$
\begin{gathered}
I\left(\lambda_{i}\right)=\int I_{0}(\lambda) \exp \left[-\sigma_{j}(\lambda) n_{j}\right] S\left(\lambda-\lambda_{i}\right) d \lambda \text { sum over } j \\
L\left(\lambda_{i}\right)=\log \left[I\left(\lambda_{i}\right)\right]
\end{gathered}
$$

Definition of $\log (): \operatorname{dlog}(x)=d x / x$

$$
\frac{\partial L\left(\lambda_{i}\right)}{\partial n_{k}}=\frac{1.0}{1\left(\lambda_{i}\right)} \frac{\partial}{\partial \eta_{k}} \int I_{0}(\lambda) \exp \left[-\sigma_{j}(\lambda) \eta_{j}\right] S\left(\lambda-\lambda_{i}\right) d \lambda
$$

## Analytic Derivatives

Discrete v. Analytic Derivatives
Discrete: $\frac{\partial y}{\partial x}=[y(x+\Delta x)-y(x)] / \Delta x$
The problem: if $\Delta x$ is too big the slope isn't appropriate to $\mathrm{y}(\mathrm{x})$

If $\Delta x$ is too small the differential is all noise.
The analytic approach avoids this.
It also calculates everything in one pass of the forward model instead of $\mathrm{N}+1$ passes.

## Result...

$$
\begin{aligned}
\frac{\partial L\left(\lambda_{i}\right)}{\partial n_{k}} & =\frac{1.0}{1\left(\lambda_{i}\right)} \frac{\partial}{\partial \eta_{k}} \int I_{0}(\lambda) \exp \left[-\sigma_{j}(\lambda) \eta_{j}\right] S\left(\lambda-\lambda_{i}\right) d \lambda \\
& =\frac{1.0}{1\left(\lambda_{i}\right)} \int I_{0}(\lambda) \frac{\partial}{\partial \eta_{k}}\left\{\exp \left[-\sigma_{j}(\lambda) \eta_{j}\right]\right\} S\left(\lambda-\lambda_{i}\right) d \lambda \\
& =-\frac{1.0}{I\left(\lambda_{i}\right)} \int I_{0}(\lambda) \sigma_{k}(\lambda) \exp \left[-\sigma_{j}(\lambda) \eta_{j}\right] S\left(\lambda-\lambda_{i}\right) d \lambda
\end{aligned}
$$

Remember?
$L\left(\lambda_{i}\right)=\log \left\{\int I_{0}(\lambda) \exp \left[-\sigma_{i}(\lambda) \eta_{j}\right] S\left(\lambda-\lambda_{i}\right) d \lambda\right\}$
So many of the terms in the differentials are already being calculated for the intensity spectrum.

## Numerology

We don't actually measure an infinite number of wavelengths So the integral form doesn't really apply:
$I\left(\lambda_{i}\right)=\int I_{0}(\lambda) \exp \left[-\sigma_{j}(\lambda) \eta_{j}\right] S\left(\lambda-\lambda_{i}\right) d \lambda$ sum over $j$
So the equation becomes

$$
I\left(\lambda_{i}\right)=\sum_{i=0}^{I<N_{\lambda}} l_{0}\left(\lambda_{i}\right) \exp \left[-\sigma_{j}\left(\lambda_{l}\right) \eta_{j}\right] S\left(\lambda_{l}-\lambda_{i}\right)
$$

sum over j

And a similar expression for the differential:

$$
\frac{\partial l\left(\lambda_{\mathrm{i}}\right)}{\partial \eta_{k}}=\left.\sum_{\mathrm{I}=0}^{\mathrm{I}<N_{\lambda}}\right|_{o}\left(\lambda_{l}\right) \sigma_{\mathrm{k}}\left(\lambda_{\mathrm{l}}\right) \exp \left[-\sigma_{j}\left(\lambda_{\mathrm{l}}\right) \eta_{j}\right] S\left(\lambda_{\mathrm{l}}-\lambda_{\mathrm{i}}\right)
$$

Where the spacing of $\lambda_{I}$ is small enough for an accurate result

## Practical

- The preceding development did not consider noise
- But different parts of the spectrum have different noise
- It is also desirable to estimate the uncertainty in the result
- Rather than minimizing the square error, one can use $\mathrm{X}^{2}$

$$
\begin{aligned}
& \text { Let } \quad \Delta_{i}=\log \left[\operatorname{Im}\left(\lambda_{i}\right)\right]-\log \left[I\left(\lambda_{i}\right)\right] \\
& X^{2}=\sum_{i=0} \sum_{j=0} \Delta_{i} \sigma_{i j}^{-1} \Delta_{j}
\end{aligned}
$$

$\sigma$ is the error covariance matrix and is frequently taken to be diagonal i.e.: in this case the individual wavelength's noise is considered to be uncorrelated

## Comments

Easy to show that if the noise on the observations is well represented by $\sigma$, then each degree of freedom (e.g. each $x_{j}$ ) should multiply to 1.0 so that $X^{2}$ will have a value equal to the number of degrees of freedom.

Scaling by $\sigma^{-1}$ normalizes the scatter of each of the dimensions of the $\mathrm{x}_{\mathrm{ij}}$. The departure of the value of $\mathrm{X}^{2}$ from the number of degrees of freedom is a measure of how well the model fits the data.

## More Advanced Methods

It is possible to generate $\mathrm{X}^{2}$ terms that constraint solutions in other ways for example:

- Prior knowledge of mean and covariance
- Smoothing terms with uncertainty matrix
- Functional constraints

This is the basis of the Rodgers optimal estimation technique
The individual terms are all of order unity per degree of freedom and can be summed and simultaneously minimized

Wavelength shift and stretch can also be simultaneously retrieved

## Other Independent Variables

Wavelength shift and Stretch:

$$
\begin{aligned}
& \lambda^{\prime}=\lambda+\text { shift }+ \text { stretch }^{*}\left(\lambda-\lambda_{c}\right) \\
& \partial \mathrm{I} / \partial \text { shift }=\partial \mathrm{I} / \partial \lambda \\
& \partial \mathrm{I} / \partial \text { stretch }=\partial \mathrm{I} / \partial \lambda\left(\lambda-\lambda_{c}\right)
\end{aligned}
$$

$\partial I / \partial \lambda$ can be determined by differentiating the interpolation routine used to interpolate $I(\lambda)$
Cross-section temperature effects:
$\sigma(\lambda, T)=\sigma\left(\lambda, T_{0}\right)+\partial \sigma / \partial T\left(T-T_{0}\right)$
$I=I_{0} \exp (-\sigma(\lambda, T) X) ; \log (I)=\log \left(I_{0}\right)-\sigma(\lambda, T) X$
$\log (I)=\log \left(I_{0}\right)-\underline{\sigma\left(\lambda, T_{0}\right)} \underline{X}-\underline{\partial \sigma / \partial T}\left(T-T_{0}\right) X$

## Conclusion

Any instrument will respond to a number of inputs
e.g. light from the wrong wavelengths thermal and pressure effects thermally generated photons non-linearity mechanical vibration or stress signal 'memory' sensitivity drift radiation effects

The experimentalist's job is to isolate the effects, model them properly and use as few retrievable parameters as possible to account for them. Ideally the 'raw' data should be fitted with a model.

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The End. Thank you.

Reaching for Space -- Why Turkeys can't fly

## Lessons One

1. Thou shalt NOT build computers. IBM can lose money at it, and you can use up ALL your resources accomplishing the goal of delivering late, a computer which was built from obsolete components. The OEM's corner the market on the up-to-date stuff.
2. Don't run anything on batteries. If there is an outlet -- plug in to it. If there is no outlet, have one installed.
3. Every instrument needs a parent. Don't let one drift along as an orphan -- it will wind up as a juvenile delinquent and embarrass you later on.
4. If a project appears to be difficult, either cancel it or provide the resources needed for it. Half-assed management leads to a half-assed project.
5. Almost ANY amount of paperwork is easy compared to making more hardware [see 1]. (i.e.: if a computer already exists -- use it.)
6. Never make a half-dozen, make a dozen. Especially if you can avoid making bits you don't have to make.

## Lessons 1 con't

7. Don't use matrix management without accountability. People see things as important if they are personally involved -- one way or another.
8. Above all - don't say yes to running a project over which you have no control!
9. If you are responsible for a project someone else is managing, try to remove obstacles from the path -- don't add sand traps and rough ground.
10. If you change managers, especially if you do so because of perceived problems in the development process, do the whole design over again. Salvage what you can, but be sure that there were no endemic problems which lead to the superficial symptoms.
11. Use benchmarks along the way to develop schedules, and then meet some of the deadlines along the way. Otherwise nothing gets done. Don't manage part of the project, or part of the time of the involved principals, because the other demands on the people will make it impossible to meet deadlines.

## Lessons Two

Don't do space experiments; but if you do...
Be somewhat conservative in technology steps
To quote Jim Drummond: Test as you fly (the whole thing!)
Recognize the difference between:

- Functionality tests
- Tests showing contractual compliance
- Tests showing science output
- Characterization

If an unplanned procedure has to be implemented have all the stakeholders on site for the operation. Different folks see different things.

Engineering is the art of never again making a mistake you have already made before. Space experiments are harder.

